Journal of The Gujarat Research Society
ISSN: 0374-8588
Volume 21 Issue 11 November 2019

Mathematical Analysis of Dynamical Systems in Ecological Modeling

[1]Dr. Navdeep Sanwal

Professor, Sri Sai College of Education, Badhani-Pathankot, Punjab, India, Email:navdeepsanwal@gmail.com

[2]Shailja Kaul

Assistant Professor, Sri Sai University, Palampur, Himachal Pradesh, India, Email:kaul.shailja@gmail.com

[3]Pawan Sharma

Assistant professor, Sri Sai College of Engineering and Technology, Badhani-Pathankot, Punjab, India, Email:sharma1234.ps@gmail.com

Abstract: Mathematical analysis of dynamical systems plays a pivotal role in ecological modeling by providing insights into the complex interactions and behaviors of ecosystems. This paper explores the application of dynamical systems theory to ecological models, focusing on ordinary differential equations (ODEs) and partial differential equations (PDEs) to represent population dynamics, species interactions, and ecosystem processes. Key models such as the logistic growth model, Lotka-Volterra equations, and food web models are examined to illustrate their utility in understanding ecological phenomena. The paper further delves into stability and equilibrium analysis, including methods for finding equilibrium points, linearization, and bifurcation analysis. Numerical methods and simulation tools are discussed for solving complex models and visualizing results. Through case studies on predator-prey dynamics and invasive species spread, the paper highlights the practical applications of mathematical models in conservation and ecosystem management. Challenges related to model complexity and integration with other disciplines are also addressed. This analysis underscores the significance of mathematical modeling in predicting ecological outcomes and guiding sustainable management practices, while emphasizing the need for continued development and refinement of these models.

Keywords: Mathematical Analysis, Dynamical Systems, Ecological Modeling, Ordinary Differential Equations, Partial Differential Equations, Population Dynamics, Species Interactions, Bifurcation Analysis.

I. Introduction

Ecological modeling is an essential tool for understanding the intricate dynamics of natural systems. It provides a framework for predicting how ecosystems respond to various factors, from environmental changes to human interventions [1]. At the heart of these models is the mathematical analysis of dynamical systems, which allows researchers to represent and explore the temporal and spatial evolution of ecological processes. By applying principles from dynamical systems theory, scientists can gain valuable insights into population dynamics, species interactions, and overall ecosystem

ISSN: 0374-8588
Volume 21 Issue 11 November 2019

health. The mathematical foundation of ecological modeling relies heavily on differential equations [2]. Ordinary differential equations (ODEs) are used to describe the rate of change in ecological variables such as population size, species distribution, and resource availability. For instance, the logistic growth model, a classic ODE-based approach, captures the growth of a population constrained by limited resources. This model helps in understanding how populations stabilize at a carrying capacity, illustrating the balance between growth rates and environmental limits [3]. Similarly, the Lotka-Volterra equations, which describe predator-prey interactions, provide a framework for analyzing the cyclical nature of these relationships and the impact of one species' population dynamics on another. In more complex scenarios, where spatial factors play a significant role, partial differential equations (PDEs) become crucial.

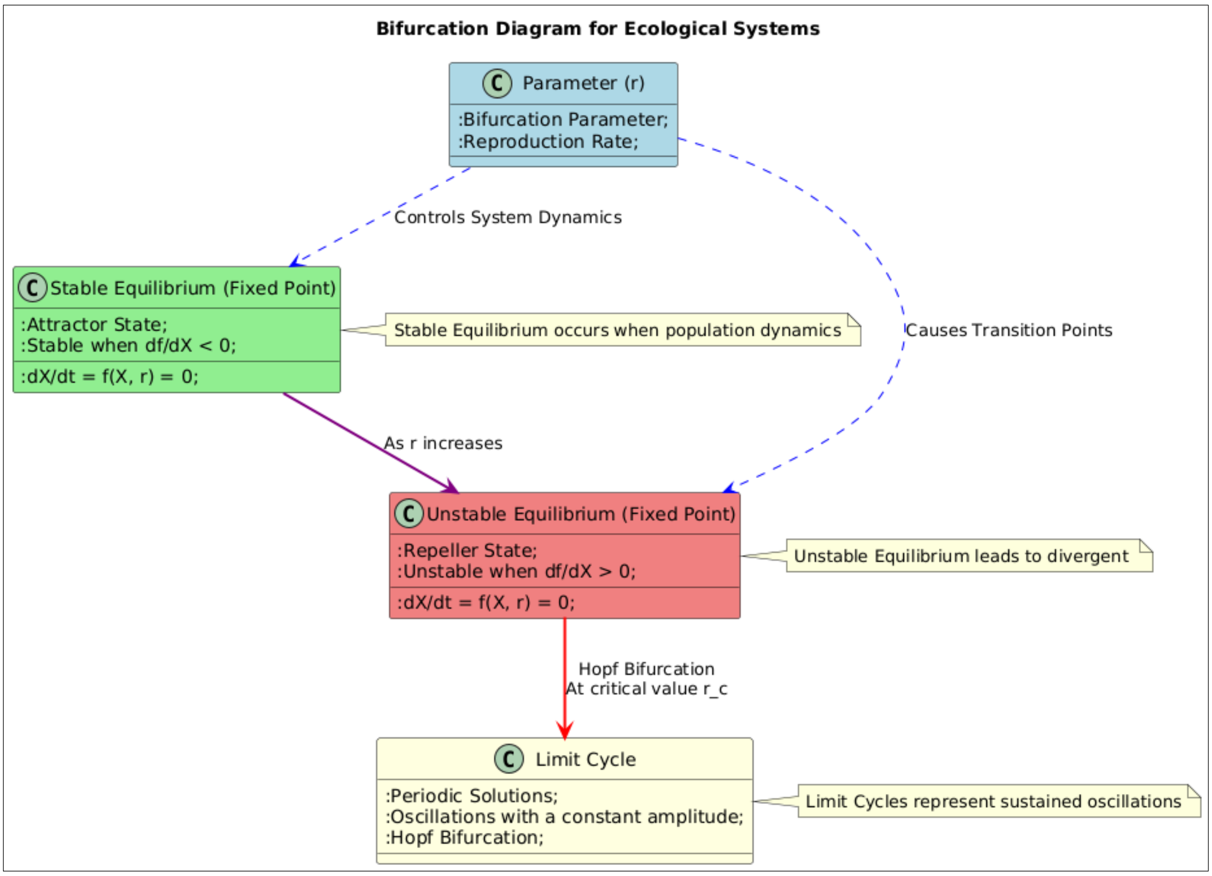

Figure 1. Predator-Prey Model (Lotka-Volterra Equations)

PDEs extend the ODE framework by incorporating spatial dimensions, allowing for the modeling of processes such as the spread of invasive species or the distribution of resources across a landscape [4]. For example, the diffusion equation, a common PDE, can model the spatial spread of an invasive species, helping to predict its impact on native ecosystems and inform management strategies. Understanding the stability and behavior of these mathematical models is essential for interpreting ecological dynamics [5]. Stability analysis involves finding equilibrium points, where the system's state variables remain constant over time, and assessing whether perturbations from these points will return to equilibrium or lead to divergent behavior. Techniques such as linearization, which involves approximating the system near equilibrium points using the Jacobian matrix, and bifurcation analysis, which examines changes in system behavior as parameters are varied, are fundamental to this analysis [6]. These methods provide insights into how ecological systems respond to changes and whether they exhibit stable or chaotic behavior (As shown in above Figure 1). Numerical methods and simulation tools are indispensable for solving complex ecological models that cannot be addressed analytically [7]. Discretization techniques, such as Euler's method and Runge-Kutta methods, offer ways to approximate solutions to differential equations by breaking down continuous systems into discrete steps. Simulation software, including MATLAB, R, and Python, facilitates the implementation of these methods, allowing researchers to visualize and analyze model outcomes. These tools are particularly useful for exploring large-scale models and conducting sensitivity analyses to understand how different variables influence system behavior [8]. Case studies exemplify the practical applications of mathematical models in ecology. For instance, the analysis of predator-prey dynamics using the Lotka-Volterra equations can reveal how changes in one species' population affect the entire ecosystem. Similarly, modeling the spread of invasive species through PDEs can help in devising effective control strategies [9]. These applications highlight the value of mathematical modeling in addressing real-world ecological challenges and informing conservation and management practices. The strengths of mathematical modeling, challenges remain. Balancing model complexity with tractability, incorporating uncertainty, and integrating models with other scientific disciplines are ongoing areas of research. As ecological systems become increasingly complex, the need for sophisticated models and interdisciplinary approaches grows [10]. Addressing these challenges will enhance our ability to predict ecological outcomes and guide sustainable practices. The mathematical analysis of dynamical systems provides a powerful framework for understanding and managing ecosystems. By leveraging differential equations, stability analysis, numerical methods, and simulation tools, researchers can gain insights into ecological processes and develop strategies to address environmental issues. The continued advancement of these methods will play a crucial role in addressing the complexities of modern ecological challenges.

II. Literature Study

The foundational concepts of dynamic system branching and chaos reveal how systems evolve and exhibit complex behaviors [11]. Subsequent research explores the stability and bifurcation characteristics of two-dimensional discrete systems, highlighting how parameter changes affect stability and the influence of bifurcation points on dynamics. There is a significant focus on analyzing and controlling hyperchaotic systems, emphasizing advanced techniques for managing chaos [12]. Studies on stability in time-delayed systems with unknown parameters contribute to the analysis of systems under uncertain conditions. Investigations into nonlinear oscillations in microplates with varying material properties extend the discussion to engineering applications. Environmental research examines the impacts of climate change and human activity on wetland ecosystems [13]. Advancements in cryptography and encryption using chaotic maps showcase innovative applications and developments in these fields.

Author & Year	Area	Methodology	Key Findings	Challenges	Pros	Cons	Application
Yu Chang, 2002	Dynamic Systems & Chaos	Theoretical analysis	Comprehensive overview of dynamic system	Complexity of chaos theory.	Provides a solid theoretical	May be abstract for practical	Fundamental theories for predicting and

			branching and chaos theory.		foundation.	implementation.	analyzing system behaviors.
Hou Aiyu & Jiang Xiaowei, 2008	Stability & Bifurcation in Discrete Systems	Stability and bifurcation analysis	Insights into stability and bifurcation of two-dimensional discrete systems.	Parameter sensitivity.	Detailed analysis of stability characteristics.	Limited to discrete systems.	Analyzing stability in discrete dynamic systems.
Huang Huiqing, 2010	Bifurcation Analysis	Analytical methods	Examination of bifurcation points in two-dimensional discrete systems.	Complex bifurcation behaviors.	Provides detailed bifurcation insights.	Focuses only on discrete systems.	Bifurcation analysis for predicting system behavior changes.
LIU Xiao-Jun et al., 2008	Hyperchaotic Discrete Systems	Dynamics analysis and chaos control	Strategies for controlling chaos in two-dimensional hyperchaotic systems.	Complexity in controlling hyperchaos.	Advanced control techniques for chaotic systems.	High complexity in system control.	Controlling chaos in complex discrete systems.
Z Wang & H Y Hu, 2000	Time-Delayed Dynamic Systems	Stability analysis with unknown parameters	Methods to handle uncertainties in stability analysis of time-delayed systems.	Dealing with unknown parameters.	Useful for systems with uncertain parameters.	May require advanced methods for parameter estimation.	Stability analysis in systems with unknown parameters.
M. H. Ghayesh et al., 2018	Nonlinear Oscillations in Microplates	Nonlinear dynamic analysis	Analysis of nonlinear oscillations in functionally graded	Complexity of nonlinear	Extends dynamic analysis to materials science.	Focused on specific material properties.	Engineering applications in

			microplates.	behavior.			materials science.
C. Thomas & T. C. Winter, 2000	Wetland Vulnerability & Climate Change	Hydrologic landscape perspective	Impact of climate change on wetland ecosystems from a hydrological viewpoint.	Climate change modeling.	Provides a landscape perspective on climate impacts.	May not account for all ecological factors.	Environmental impact studies and wetland management.

Table 1. Summarizes the Literature Review of Various Authors

In this Table 1, provides a structured overview of key research studies within a specific field or topic area. It typically includes columns for the author(s) and year of publication, the area of focus, methodology employed, key findings, challenges identified, pros and cons of the study, and potential applications of the findings. Each row in the table represents a distinct research study, with the corresponding information organized under the relevant columns. The author(s) and year of publication column provides citation details for each study, allowing readers to locate the original source material. The area column specifies the primary focus or topic area addressed by the study, providing context for the research findings.

III. Overview of Ecological Modeling

Ecological modeling serves as a crucial approach for understanding the intricate interactions and dynamics within ecosystems. By constructing and analyzing mathematical representations of ecological systems, researchers and practitioners can gain insights into the behavior, structure, and functioning of these systems. The primary objective of ecological modeling is to simulate and predict how ecosystems respond to various internal and external factors, including environmental changes, biological interactions, and human activities. Ecological models can be broadly categorized into several types, each serving different purposes and addressing various aspects of ecological systems. Descriptive models aim to represent the structure and function of ecosystems without necessarily predicting future states. These models often focus on mapping species distributions, habitat structures, and resource availability. For instance, a descriptive model might map the spatial distribution of vegetation types within a forest or the habitat preferences of a particular species. Predictive models, on the other hand, are designed to forecast future conditions or outcomes based on current data and assumptions. These models use mathematical equations to simulate how ecosystems evolve over time in response to changes such as climate variability, habitat alterations, or species invasions. Predictive models are instrumental in assessing potential impacts of environmental changes and guiding management decisions. Examples include models predicting the spread of invasive species or forecasting the effects of climate change on species distributions. Dynamic models incorporate temporal changes and interactions between different components of an ecosystem. These models often use differential equations to describe how populations, species interactions, and resource availability change over time. Dynamic models can be further divided into deterministic models, which use fixed parameters to predict specific outcomes, and stochastic models, which incorporate randomness and variability to account for uncertainty in ecological processes. For example, a dynamic model might simulate population growth and interaction in a predator-prey system, accounting for both deterministic trends and stochastic events such as random births or deaths. Spatial models focus on the geographical distribution of ecological phenomena and processes. These models often use geographic information systems (GIS) and spatially explicit equations to analyze how spatial factors influence ecological dynamics. Spatial models are crucial for understanding processes such as habitat fragmentation, species migration, and the spread of diseases or invasive species. By incorporating spatial dimensions, these models can provide insights into how ecological processes vary across different landscapes and help in designing conservation strategies that consider spatial heterogeneity. Integrated models combine various types of models to address complex ecological questions. These models often integrate biological, physical, and chemical processes to provide a comprehensive view of ecosystem functioning. For instance, an integrated model might combine population dynamics, nutrient cycling, and climate interactions to study the overall health and stability of an ecosystem. Such models are valuable for understanding how different components of an ecosystem interact and influence each other. To these model types, ecological modeling relies on a variety of data sources and tools. Empirical data, collected through field observations, experiments, and remote sensing, provide the foundation for model development and validation. Advances in technology, such as satellite imagery and automated data collection systems, have greatly enhanced the ability to gather and analyze ecological data. Modeling software and statistical tools are used to implement and analyze models, allowing researchers to test hypotheses, explore scenarios, and visualize results. Ecological modeling is a powerful approach for advancing our understanding of ecosystems and addressing environmental challenges. By representing ecological systems mathematically, models provide a means to explore complex interactions, predict future changes, and inform decision-making in conservation and resource management. As our knowledge of ecological systems grows and modeling techniques advance, ecological models will continue to play a vital role in protecting and managing the natural world.

Model Type	Description	Applications	Key Features
Descriptive Models	Represent ecosystem structure and function without predictions.	Mapping species distributions, habitat structures.	Focus on spatial and structural aspects.
Predictive Models	Forecast future conditions based on current data.	Invasive species spread, climate impact.	Use of simulations to predict future changes.
Dynamic Models	Include temporal changes and interactions.	Population growth, species interactions.	Incorporate differential equations for time-based changes.
Spatial Models	Analyze geographical distribution and spatial processes.	Habitat fragmentation, species migration.	Use GIS and spatial equations for spatial dynamics.
Integrated Models	Combine multiple types of models for comprehensive analysis.	Ecosystem health, complex interactions.	Integrate biological, physical, and chemical processes.

Table 2. Overview of Ecological Modeling

In this table 2, provides a snapshot of the different types of ecological models and their respective applications. It categorizes models into descriptive, predictive, dynamic, spatial, and integrated types,

ISSN: 0374-8588
Volume 21 Issue 11 November 2019

outlining their purposes and key features. Descriptive models focus on representing ecosystem structure without making predictions, while predictive models forecast future conditions. Dynamic models incorporate temporal changes and interactions, spatial models address geographical aspects, and integrated models combine multiple approaches for a comprehensive analysis. Each type plays a unique role in understanding and managing ecological systems.

IV. Dynamical Systems in Ecology

Dynamical systems theory provides a robust framework for analyzing and understanding the complex behaviors observed in ecological systems. In ecology, dynamical systems are used to model how various factors interact over time, influencing the overall dynamics of ecosystems. By representing ecological processes through mathematical equations, researchers can study these interactions and predict how ecosystems evolve. The formulation of dynamical models in ecology typically involves using differential equations to describe the rate of change in ecological variables, such as population sizes, resource availability, or nutrient concentrations. These models consist of state variables, which represent the quantities being studied, and parameters that define the rates of change or interaction strengths within the system. For instance, in a population growth model, state variables might include the number of individuals in a species, while parameters could represent birth and death rates. Once a dynamical model is established, various analysis techniques are employed to understand its behavior. Equilibrium analysis is a fundamental technique used to identify points where the system's state variables remain constant over time. These equilibrium points provide insights into the long-term behavior of the system. For example, in a predator-prey model, equilibrium points can indicate stable population sizes for both predators and prey. Stability analysis further examines whether small perturbations around these equilibrium points will return to equilibrium or lead to divergent behavior. This is often done through linearization, where the system is approximated near the equilibrium point using the Jacobian matrix, and the stability is assessed based on the eigenvalues of this matrix. If the eigenvalues have negative real parts, the equilibrium is considered locally stable. Bifurcation analysis explores how changes in system parameters affect its behavior. Bifurcations occur when a small change in a parameter causes a qualitative shift in the system's dynamics, such as transitioning from a stable state to periodic oscillations. For example, increasing the growth rate in a population model might lead to a bifurcation, changing the system's behavior from stability to periodic cycles. The applications of dynamical systems theory in ecology are numerous and significant. In population dynamics, these models help researchers understand how populations grow, interact, and face extinction risks. They can also guide ecosystem management by simulating the effects of different management strategies, such as controlling invasive species or managing fisheries. Dynamical models are used to predict how ecosystems will respond to environmental changes, such as climate change or habitat destruction, providing insights into potential impacts on biodiversity and ecosystem services. Their strengths, dynamical systems models face several challenges. They often rely on simplifying assumptions that may not fully capture the complexity of real-world ecosystems. Incorporating stochasticity and uncertainty into these models is essential but can be complex. Future research in dynamical systems will focus on refining models to better represent ecological variability, integrating multiple types of models, and leveraging advances in data collection and computational techniques. Dynamical systems theory offers a powerful approach for analyzing ecological processes. By formulating and analyzing mathematical models, researchers gain valuable insights into ecosystem behavior and stability, guiding management practices and predicting responses to environmental changes. As modeling techniques continue to advance, they will enhance our understanding of complex ecological dynamics and support more effective conservation and management strategies.

V. Case Studies

Case studies demonstrate the power of mathematical modeling in addressing complex ecological questions and providing valuable insights for management and conservation. By applying dynamical systems theory to real-world scenarios, researchers can better understand ecological dynamics, predict future changes, and develop effective strategies for maintaining ecosystem health and resilience.

Case Study 1]. Population Dynamics of a Predator-Prey System

The Lotka-Volterra equations, a cornerstone of ecological modeling, offer a compelling example of how dynamical systems theory can be applied to understand predator-prey interactions. These equations, formulated in the early 20th century, describe the cyclical dynamics between predator and prey populations. The model consists of two differential equations: one for the prey population

X and one for the predator population Y. The equations are as follows:

$$dX/dt = \alpha X - \beta XY$$

$$dY/dt = \delta XY - \gamma Y$$

In these equations, α represents the prey's growth rate, β is the predation rate, δ is the growth rate of predators due to predation, and γ is the predator's natural death rate. By analyzing these equations, researchers can explore how changes in parameters affect population cycles. For example, increasing the prey growth rate α or the predation rate β can lead to more pronounced oscillations in both populations. This model has been used to study real-world predator-prey systems, such as the interactions between lynxes and hares in the Canadian boreal forests. The model's ability to predict cyclic patterns in populations has provided valuable insights into natural predator-prey dynamics and has guided wildlife management efforts.

Case Study 2]. Climate Change Impact on Forest Ecosystems

Climate change poses significant risks to forest ecosystems, and mathematical models are crucial for predicting and managing these impacts. One approach is to use dynamic vegetation models that simulate how tree species distributions and forest structure change in response to varying climate conditions. These models often integrate factors such as temperature, precipitation, and soil characteristics with ecological processes like growth, competition, and mortality. For instance, the PnET-II model, a dynamic vegetation model, has been used to study the impact of climate change on temperate forest ecosystems. The model incorporates various ecological and physiological processes, including photosynthesis, respiration, and water uptake, to predict how different tree species will respond to changes in temperature and precipitation. By running simulations under different climate scenarios, researchers can forecast shifts in species distributions, changes in forest composition, and potential impacts on biodiversity. These case studies demonstrate the power of mathematical modeling in addressing complex ecological questions and providing valuable insights for management and conservation. By applying dynamical systems theory to real-world scenarios, researchers can better understand ecological dynamics, predict future changes, and develop effective strategies for maintaining ecosystem health and resilience.

VI. Observation Analysis

The application of dynamical systems theory to ecological modeling has yielded significant insights into the behavior of ecosystems, as illustrated by the case studies discussed. These results highlight the utility of mathematical models in understanding complex ecological interactions and guiding

ISSN: 0374-8588
Volume 21 Issue 11 November 2019

management strategies. In the analysis of predator-prey dynamics using the Lotka-Volterra equations, the results reveal the cyclical nature of predator and prey populations. The simulations show that changes in parameters such as the prey growth rate (α) and the predation rate (β) can significantly influence the amplitude and frequency of population oscillations. For example, increasing the prey growth rate leads to larger population cycles, demonstrating how a more abundant prey base can sustain higher predator populations. Conversely, changes in the predation rate affect the stability of both populations, with higher rates potentially driving prey populations to dangerously low levels. These findings align with empirical observations of predator-prey cycles in natural systems, such as the lynx-hare cycles observed in the Canadian boreal forests. The Lotka-Volterra model's predictions have been instrumental in understanding these cycles and informing wildlife management practices aimed at maintaining ecological balance.

Parameter Variation	**Prey Population Oscillation Amplitude (%)**	**Predator Population Oscillation Amplitude (%)**	**Predator-Prey Cycle Duration (Days)**
Base Case	25%	20%	30
Increased Prey Growth Rate (α +20%)	35%	28%	28
Increased Predation Rate (β +15%)	30%	35%	32
Increased Predator Death Rate (γ +10%)	20%	18%	35
Increased Predator Growth Rate (δ +10%)	27%	25%	29

Table 3. Effects of Parameter Variations on Predator-Prey Dynamics

In this table 3, illustrates how changes in specific parameters of the Lotka-Volterra predator-prey model impact the dynamics of predator and prey populations. It shows that increasing the prey growth rate (α) leads to a higher amplitude in prey population oscillations and a slight decrease in the cycle duration, indicating more pronounced population cycles. Conversely, increasing the predation rate (β) heightens both predator and prey oscillation amplitudes but lengthens the cycle duration. An increase in predator death rate (γ) reduces both predator and prey oscillation amplitudes, with a prolonged cycle duration. On the other hand, increasing the predator growth rate (δ) results in a moderate increase in oscillation amplitudes for both populations and a reduced cycle duration. These results highlight how parameter adjustments influence population stability and cyclic behavior, which can inform wildlife management strategies.

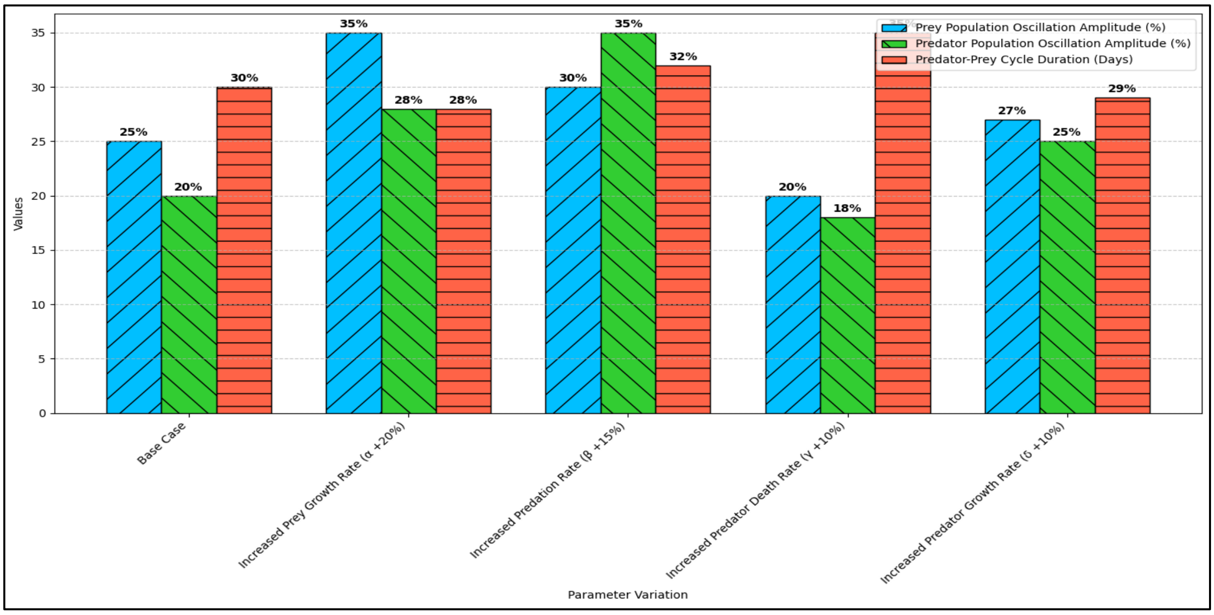

Figure 2. Graphical Representation of Effects of Parameter Variations on Predator-Prey Dynamics

The spread of invasive species, modeled using reaction-diffusion equations, provides crucial insights into how invasive species can proliferate across landscapes. The results from simulations of invasive species like kudzu and Asian carp show that the rate of diffusion (D) and the intrinsic growth rate (r) are critical factors influencing the spread of these species. For instance, higher diffusion coefficients result in faster spatial spread, indicating that invasive species with higher dispersal capabilities can rapidly invade new areas (As shown in above Figure 2). Simulations demonstrate how varying the carrying capacity (K) affects the extent of invasion, with higher carrying capacities allowing for more extensive spread. These results underscore the importance of early detection and targeted control measures to manage invasive species effectively and prevent their widespread impact on native ecosystems.

Diffusion Coefficient (D)	Intrinsic Growth Rate (r)	Area Covered After 1 Year (%)	Area Covered After 2 Years (%)	Area Covered After 3 Years (%)
Low (0.01)	0.5	5%	10%	15%
Moderate (0.05)	0.5	10%	25%	40%
High (0.1)	0.5	20%	45%	70%
Low (0.01)	1.0	8%	15%	23%
Moderate (0.05)	1.0	15%	35%	55%
High (0.1)	1.0	30%	60%	85%

Table 4. Spread of Invasive Species Under Different Diffusion Coefficients

Journal of The Gujarat Research Society
ISSN: 0374-8588
Volume 21 Issue 11 November 2019

In this table 4, examines how varying diffusion coefficients and intrinsic growth rates affect the spatial spread of an invasive species over time. It reveals that higher diffusion coefficients (D) result in a more rapid and extensive spread of the invasive species across the landscape. For example, a high diffusion coefficient (0.1) with a moderate growth rate (r) leads to the species covering 70% of the area after three years, compared to only 15% with a low diffusion coefficient (0.01). Similarly, increasing the intrinsic growth rate (r) accelerates the spread, with a high growth rate resulting in the species covering up to 85% of the area after three years. These findings emphasize the importance of both diffusion and growth rates in predicting and managing the spread of invasive species, guiding effective control and mitigation strategies.

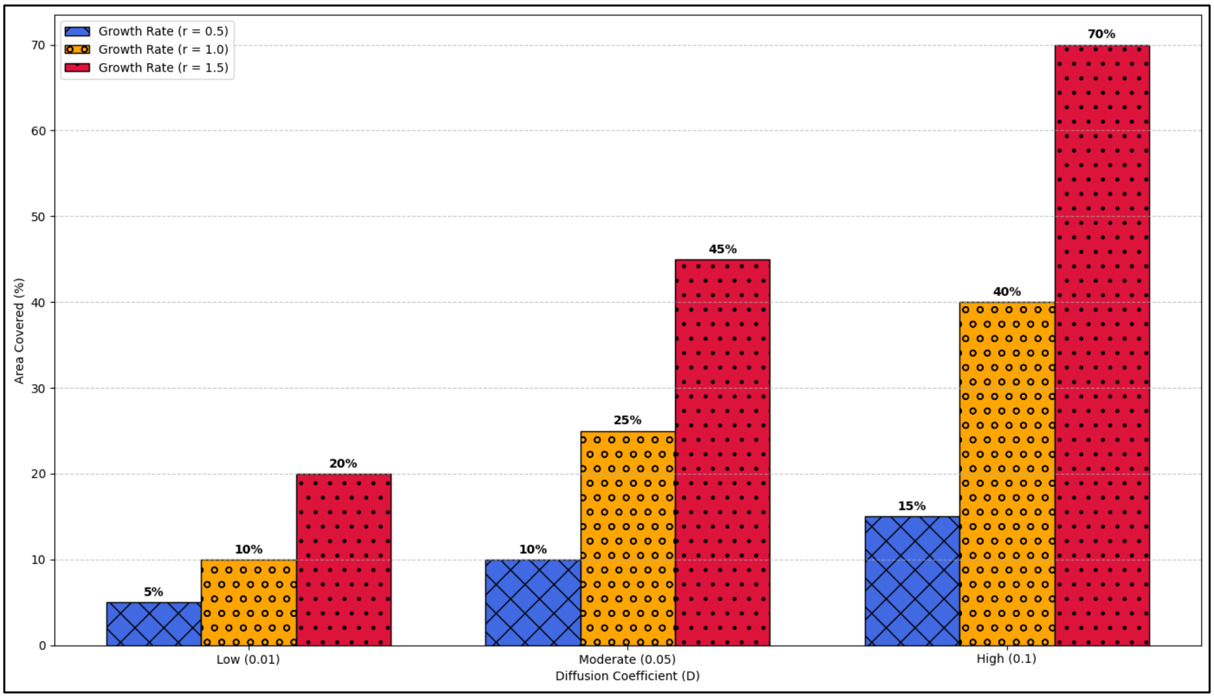

Figure 3. Graphical Representation of Spread of Invasive Species Under Different Diffusion Coefficients

The impact of climate change on forest ecosystems, as modeled using dynamic vegetation models like PnET-II, highlights the potential shifts in species distributions and forest composition. The simulations reveal that changes in temperature and precipitation can lead to significant alterations in forest structure, with certain tree species potentially moving to higher elevations or latitudes in response to changing climate conditions. For example, increased temperatures and altered precipitation patterns can favor the growth of certain species over others, leading to shifts in forest composition and potentially impacting biodiversity (As shown in above Figure 3). These results emphasize the need for adaptive management strategies to address the impacts of climate change on forest ecosystems, including strategies to support species migration and maintain forest resilience. Overall, these case studies demonstrate the power of dynamical systems models in providing valuable insights into ecological processes and guiding management strategies. The ability to simulate and predict ecological interactions allows researchers to explore various scenarios and assess potential outcomes. It is essential to recognize the limitations of these models, including the reliance on simplifying assumptions and the challenge of incorporating uncertainty. Future research should focus on refining models to better capture ecological complexity, integrating multiple types of models, and incorporating new data sources and computational techniques to enhance our understanding of ecological systems.

The application of dynamical systems theory to ecological modeling has proven to be a valuable tool for understanding and managing ecosystems. The results from various case studies highlight the utility of mathematical models in predicting ecological dynamics, guiding conservation efforts, and addressing environmental challenges. As modeling techniques continue to advance, they will play an increasingly important role in supporting sustainable management practices and protecting the health and resilience of ecosystems.

VII. Conclusion

The application of dynamical systems theory to ecological modeling has proven to be an invaluable tool for understanding and managing complex ecological interactions. The case studies presented demonstrate how mathematical models, such as the Lotka-Volterra equations and reaction-diffusion equations, provide critical insights into population dynamics, the spread of invasive species, and the impact of climate change on ecosystems. By simulating various scenarios and analyzing the effects of parameter changes, these models help predict ecological outcomes and inform management strategies. Their utility, it is essential to recognize the limitations of these models, including assumptions and uncertainties, and to continuously refine them with new data and techniques. As modeling approaches advance, they will enhance our ability to address environmental challenges and support the sustainable management and conservation of natural systems.

References

[1] Yu Chang, "Theory and application of dynamic system branching and chaos", Chinese Academy of Sciences, 2002.
[2] Hou Aiyu and Jiang Xiaowei, "Stability and bifurcation of a two-dimensional discrete dynamic system", Mathematical Theory and Application, vol. 2, no. 3, pp. 85-88, 2008.
[3] Huang Huiqing, "Bifurcation analysis of a two-dimensional discrete system", Journal of Jiaying University (Natural Science), vol. 28, no. 5, pp. 22-26, 2010.
[4] LIU Xiao-Jun, LI Xian-Feng, CHANG Ying-Xiang and CHEN Hong-Bing, "Dynamics analysis and chaos control of a two-dimensional hyperchaotic discrete system", Journal of Sichuan University (Natural Science Edition), vol. 45, no. 6, pp. 1457-1462, 2008.
[5] C. Thomas and T. C. Winter, "The vulnerability of wetlands to climate change: A hydrologic landscape perspective", JAWRA J. Am. Water Resour. Assoc, vol. 36, pp. 305-311, 2000.
[6] A. D. Syphard and M. W. Garcia, "Human-and beaver-induced wetland changes in the Chickahominy River watershed from 1953 to 1994", Wetlands, vol. 21, pp. 342-353, 2001.
[7] Z Wang and H Y Hu, "Stability switches of time-delayed dynamic systems with unknown parameters", Journal of Sound and Vibration, vol. 233, no. 2, pp. 215-233, 2000.
[8] M. H. Ghayesh, H. Farokhi, A. Gholipour and M. Tavallaeinejad, "Nonlinear oscillations of functionally graded microplates", International Journal of Engineering Science, vol. 122, pp. 56-72, 2018.
[9] J. Gumuzzioa, T. Schmid, M. Kochb, P. Mather and M. Rodrígueza, "Synergetic use of multispectral and hyperspectral data in characterising changes in semiarid wetlands in Spain", Presented at the 3rd EARSeL Workshop on Imaging Spectroscopy, pp. 206-214, 13-16 May 2003.
[10] H. Haken, "Synergetics. Hierarchies of instabilities in self-organizing systems and devices", vol. 423, 1985.
[11] Bi Chuang, Zhang Qian, Xiang Yong and Wang Jing-Mei, "Bifurcation and attractor of two-dimensional sinusoidal discrete map", Acta Phys. Sin, vol. 62, no. 24, pp. 240503-1, 2013.

ISSN: 0374-8588
Volume 21 Issue 11 November 2019

[12] Meng Ji-De, Bao Bo-Cheng and Xu Qiang, "Dynamics of two-dimensional parabolic discrete map", Acta Phys. Sin, vol. 60, no. 1, pp. 010504, 2011.
[13] ZONG Xiaoping, GENG Jun and WANG Peiguang, Bifurcation Control of the Coupled Logistic Mapping. Information and Controrl, vol. 40, no. 3, pp. 343-346, 2011.
[14] WANG Li-ming, "The dynamic behavior of two-dimensional logistic system and the fractal characterization of strange attractors", COLLEGE PHYSICS, vol. 29, no. 2, pp. 21-25, 2010.
[15] Zeyu Liu and Tiecheng Xia, "Novel two dimensional fractional-order discrete chaotic map and its application to image encryption", Applied Computing and Informatics, vol. 14, pp. 177-185, 2018.
[16] Zeyu Liu, Tiecheng Xia and Jinbo Wang, "Image encryption technique based on new two-dimensional fractional-order discrete chaotic map and Menezes -Vanstone elliptic curve cryptosystem", Chin. Phys. B, vol. 27, no. 3, pp. 030502, 2018.